Accuracy and reliability of tropospheric wet refractivity tomography with GPS, BDS, and GLONASS observations

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Received 25 July 2017; received in revised form 3 December 2017; accepted 11 January 2018
Available online 31 January 2018

Abstract

Tropospheric tomography has been developed as a promising tool with which to model the three-dimensional (3-d) spatio-temporal distribution of water vapour field using Global Positioning System (GPS) observations. With the development of multi-GNSS, the accuracy and reliability of tomography using multi-GNSS observations remains to be verified. The observed multi-GNSS (GPS, GLONASS, and BDS) data, are first used to validate the tomographic results derived from various multi-GNSS combined strategies and compared with radiosonde data. In addition, the top boundary of the tomography area is determined based on the variations of water vapour with altitude derived from the COSMIC RO profiles for the period from 2006 to 2016. Tomography experiments were carried out with the 7-day data from six stations in the CORS Network at Guiyang, China. Numerical results reveal that the use of multi-GNSS data can increase the number of satellite rays used and consequently improve the coverage rate of voxels crossed by rays. Tomographic results also show that multi-GNSS observations can increase the accuracy of 3-d wet refractivity reconstruction but not as well as was expected when using currently available techniques.

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Keywords: GNSS; Tropospheric tomography; Wet refractivity; COSMIC

1. Introduction

The ability to sense atmospheric water vapour information has been proved by using Global Navigation System (GPS) observations (Bevis et al., 1992, 1994; Flores et al., 2000; Troller, 2004; Rohm and Bosy, 2009). Currently, acquiring the Precipitable Water Vapour (PWV) based on multi-GNSS data, including GPS, the BeiDou navigation satellite system (BDS), GLONASS, and Galileo, has been realised (Lu et al., 2016). Mendonça et al. (2016) has obtained the PWV value to an accuracy of about 1.7 mm based on combined GPS and Galileo observations. The corresponding PWV values were also estimated based on GPS/GLONASS observations and GPS/BDS observations with accuracies of 1.5–2.3 mm, and 1.3–1.8 mm, respectively, by Lu et al. (2015, 2016). Although the PWV values can be obtained with a high precision based on multi-GNSS observations, they cannot reflect the three-dimensional variation of water vapour concentrations, because the estimated PWV is the mean value of slant water vapour (SWV) from GNSS signals with different elevation angles and azimuths. Therefore, the three-dimensional wet refractivity cannot be obtained merely based on multi-GNSS PWV, but needs another technique (e.g., tomography) to assist in the process as well.

Tomography, as a technique, was first introduced by Radon (1917) to acquire the density distribution of an
object: it was applied to tropospheric science by Flores et al. (2000) to obtaining the three-dimensional water vapour distribution information from GPS observations. The water vapour tomography technique has many potential applications such as meteorological research and assimilation into the numerical weather models (Bennitt and Jupp, 2012; Yao et al., 2017); however, the discretised voxels of the tomography region, especially those at the lower layers, are almost empty due to the small number of satellite rays at low elevations (Bender and Raabe, 2007). A possible solution is to increase the number of satellite rays from the upcoming GNSS constellation (GLONASS, BeiDou, and Galileo). The other methods used to increase the number of filled voxels are increasing the number of GNSS stations or adjusting the horizontal resolution, or size, of the voxels relative to those used in the GNSS network (Champollion et al., 2005, Bender et al., 2011b, Benevides et al., 2017). By the end of 2016, the number of operational satellites in the GLONASS, BeiDou, and Galileo networks were 24, 16, and 20, respectively. Therefore, it is expected that more observations from multi-GNSS data can be used for tropospheric tomography. Several studies show that the increment of coverage rate of the research area crossed by rays has a positive influence on the improvement of the quality of the reconstructed water vapour field based on simulated multi-GNSS data (Bender et al., 2011a; Wang et al., 2014; Benevides et al., 2017).

By 2008, tropospheric tomography using the triple constellation (GPS, GLONASS, and Galileo) and its simulated observations was first performed by Crespi et al. (2008) based on SoTT (Software for Troposphere Tomography). Bender et al. (2011a) carried out a tomographic study for the whole of Germany with simulated GPS, Galileo, and GLONASS data from about 350 GNSS stations. The corresponding wet refractivity tomographic study was also performed by Wang et al. (2014) in China using simulated GPS, GLONASS, and BDS data: it reveals that high-precision tomographic results can be obtained with BDS system data alone. Benevides et al. (2015) proved that the Galileo can increase the accuracy and spatial resolution of GPS tomographic results based on the simulated GPS and Galileo observations, and the corresponding analysis of Galileo and GPS integration for GNSS tomography was performed (Benevides et al., 2017). Tropospheric tomography, using multi-GNSS observations, can increase the percentage of voxels crossed by rays and the spatial resolution in the research area. Therefore, the quality of the tomography is enhanced. Among the above troposphere tomography studies, one point should be noted such that the experimental data are all from simulated multi-GNSS observation; however, the data quality of simulated GNSS observations is generally better than that of any observed data. The simulated procedure was unable to simulate real errors in GNSS observations due to the fact that it usually does not consider the influences of various areas, latitude, topography, and other factors (Bender et al., 2011b; Wang et al., 2014). Therefore, the tomographic result obtained using simulated GNSS observations cannot reflect the actual three-dimensional distribution of water vapour fields.

To overcome this deficiency, this work performed a tropospheric tomography study using the observed GPS, GLONASS, and BDS data, to ascertain the advantages of tropospheric tomography in a possible future scenario involving multi-GNSS measurements. This paper is organised as follows: Section 2 describes the basic theory of tropospheric tomography, data selection and the determination of the tomographic top boundary is presented in Section 3, while the comparison of tomographic results using the data from various combined multi-GNSS observations is explained in Section 4, and conclusions and discussion thereof are presented in Section 5.

2. Theory of GNSS tropospheric tomography

2.1. GNSS observation equation for tomographic modelling

Troposphere tomography requires the probed atmospheric area to be discretised into many voxels, and it is assumed that the water vapour content of each voxel remains unchanged within a selected time period. Therefore, a ray crosses the modelling area from receiver to satellite as expressed by (Flores et al., 2000):

$$SWD = 10^{-6} \int x(s) ds = \sum d_{ijk} \cdot x_{ijk}$$

where $x$ is the wet refractivity parameter (unit: ppm) and $ds$ represents the distance of a satellite ray path element, SWD is the slant wet delay, which can be obtained based on the mapping function and the zenith wet delay (ZWD) as derived by Flores et al. (2000):

$$SWD = mf \cdot ZWD + mf \cdot \cot(e)(G_{n}^w \cdot \cos(\phi) + G_{e}^w \cdot \sin(\phi))$$

where $mf$ is the wet mapping function used in the data-processing software, $G_{n}^w$ and $G_{e}^w$ are elements of the wet delay gradient in the north-south and east-west directions, and $e$ and $\phi$ are the satellite elevation angle and azimuth angle, respectively. Therefore, a linear relationship between SWD and wet refractivity using a large number of rays can be established (Flores et al., 2000):

$$SWD_{m\times n} = A_{m\times n} \cdot N_{n\times 1}$$

where $SWD$ is a column vector with the amount of SWD observations crossing outwards from the top boundary of the research area, $A$ is a coefficient matrix with components of distance observed by the divided voxels, $N_e$ is a column vector of the wet refractivity unknowns, and $m$ and $n$ are the number of SWDs and the number of unknown parameters, respectively.
2.2. Constraint equations

Due to the geometric distribution of GNSS constellation as well as the receivers located at the research area, the observed geometry resembles an inverted cone, which is unfavourable for tropospheric tomography (Champollion et al., 2005; Benevides et al., 2015). Consequently, many discretised voxels, especially those in the lower layers, are not crossed by any satellite rays, which gives rise to an ill-conditioned design matrix (Flores et al., 2000; Champollion et al., 2009). To overcome this, a priori information, or physical constraints, are needed (Rohm, 2013; Benevides et al., 2016). In our study, two constraints are introduced: one horizontal and the other vertical. For the horizontal constraint, the wet refractivity value in a certain grid is regarded as the weighted mean value of its nearby grids in a horizontal direction (Rius et al., 1997), which can be expressed as (Guo et al., 2016):

\[
\sum_{q=1}^{q} \left( \frac{1}{C_1} N_{w_1} + \frac{1}{C_2} N_{w_2} + \cdots + \frac{1}{C_{i-1}} N_{w_{i-1}} - N_{w_1} + \frac{1}{C_{i+1}} N_{w_{i+1}} + \cdots + \frac{1}{C_n} N_{w_n} \right) = 0
\]

where \( q \) is the total number of grids in one layer, and \( w \) is the weighted horizontal coefficient, which can be calculated based on the Gaussian weighting function (Song et al., 2006).

For the vertical constraint, an exponential negative function is introduced based on the general distribution of water vapour with height (Elösegui et al., 1998). The relationship between two adjacent layers can be described by (Flores et al., 2000):

\[
N_{w_k}/N_{w_{k-1}} = a \cdot e^{-b \Delta h}
\]

where \( a \) and \( b \) are the unknown coefficients, which can be estimated with the radiosonde data from the first three days before the tomography epoch, three days of radiosonde data are selected to allow the generation of a better result, and \( \Delta h \) is the geodetic vertical height difference between two adjacent layers (unit: km). As a result, the conventional tomography model for water vapour reconstruction can be established by imposing the horizontal and vertical constraints as follows (Flores et al., 2000):

\[
\begin{pmatrix}
A \\
H \\
V
\end{pmatrix} \cdot x = \begin{pmatrix}
SWD \\
0 \\
0
\end{pmatrix}
\]

where \( H \) and \( V \) are the coefficient of horizontal and vertical constraints, respectively. To obtain the final reconstructed wet refractivity field, singular value decomposition is introduced in the inversion of Eq. (6), as presented elsewhere (Hajj et al., 1994; Ruffini et al., 1998; Press et al., 1997; Flores et al., 2000).

3. Data selection and determination of tomographic top boundary

3.1. Experimental data and processing

The data derived from six ground-based GNSS stations (as shown by ▲ in Fig. 1) in a CORS Network around Guiyang, the capital of Guizhou Province, are selected for the period from DoY 303 to 309 2015. The tomography region is determined as follows, the latitude is from 106.10° E to 107.30° E while the longitude is from 26.10° N to 27.30°, both with a horizontal resolution of 0.15° (Yao and Zhao, 2016a,b; Zhao and Yao, 2017). The vertical height of the research area is selected as 8 km, which will be discussed in Section 3.2, with the non-uniform resolution of two layers of 0.5 km, three layers of 0.6 km, four layers of 0.8 km, and two layers of 1 km, respectively. Therefore, the total number of grids is \( 8 \times 8 \times 11 \) (eight longitudinal and latitudinal, 11 vertical). In addition, there is one radiosonde station (as shown by ● in Fig. 1) located in the research area, where the radiosonde balloon is launched twice daily at UTC 00:00 and 12:00, respectively.

GNSS observations for the experimental period are processed with the independently developed multi-GNSS Precise Point Positioning (PPP) software (Zumberge et al., 1997; Kouba 2009, 2015). This software adopts the conventional ionosphere-free (IF) code and phase measurements, and the unknown parameters are estimated using an extended Kalman filter. The troposphere parameter ZTD is given at 30 s, extrapolated from the parameters interval sampling and the wet gradients in north-south and east-west directions are estimated at intervals of 2 h,
respectively. The zenith hydrostatic delay (ZHD) is calculated based on the Saastamoinen model (Saastamoinen, 1973) using the surface pressure interpolated from data for nearby grid points from the European Centre for Medium-range Weather Forecasts (ECMWF) ERA-Interim products with a resolution of 0.125° × 0.125° (Berrisford et al., 2009; Balsamo et al., 2015). Therefore, the ZWD can be obtained thus: ZWD = ZTD − ZHD, and the SWD can be calculated by using Eq. (2). The primary configuration of the multi-GNSS PPP software is described in Table 1 (Zumberge et al., 1997; Kouba 2009, 2015; Liu et al., 2017).

### 3.2. Determination of tomographic top boundary

The selection of tomographic height is a key factor for wet refractivity reconstruction (Yao and Zhao, 2016a,b). An unreasonable tomographic height can result in relatively large wet refractivity parameters or possible negative parameters, if the top boundary is too low or high. Therefore, as described in Section 3.1, the vertical height of the research area is selected as 8 km, which is determined according to the variations in wet refractivity with altitude rather than based on experience. The wet refractivity profiles of the research area are derived from the 144 “wetPrf” profiles of Constellation Observing System for Meteorology, Ionosphere, and Climate (COSMIC) Radio Occultation products from 2006 to 2016 (Sun et al., 2010; Cook et al., 2015).

Fig. 2 shows the vertical distribution of wet refractivity in a single COSMIC RO event (left) and the average values of wet refractivity for the 144 “wetPrf” profiles (right). It can be seen from the Fig. 2 that the wet refractivity is very low at 9 km above mean sea level 9 km, which is determined as the tomographic top boundary. One point should be noted that the average altitude of the selected area above mean sea level is more than 1 km, therefore, the actual height of tomographic area in our study is 8 km.

![Fig. 2](image_url)

**Fig. 2.** The vertical distribution of water vapour density in a single COSMIC RO event (left) and the 3-D water vapour distribution of tomography area and exponential fitting curve from 2006 to 2016 (right) for the selected area, X is the Mean Sea Level (MSL) altitude.
4. Comparison of tomographic results using various combined multi-GNSS observations

To assess the quality of tomographic result from multi-GNSS measurements, six combined strategies, using multi-GNSS data, are generated as follows: GPS (G), BDS(C), GLONASS (R), GPS+BDS (G+C), GPS+GLONASS (G+R) and GPS+BDS+GLONASS (G+C+R). Two groups of comparisons are carried out in our study, one is a comparison in a single day (DoY 303, 2015) with a tomography step of 1 h, the other is a comparison of tomographic result with radiosonde data at two epochs of UTC 00:00 and 12:00 for the period of seven days (DoY 303 to 309, 2015). The tomography is performed with a step of 1 h using the SWDs at 10 min intervals, and the tomography modelling is resolved based on the singular value decomposition (SVD) method (De Lathauwer et al., 1994). As described in Section 2.2, the radiosonde data from the first three days before the tomography epoch are used to calculate parameters $a$ and $b$ in Eq. (5).

4.1. Validation in a single day

The number of satellite rays used and the coverage rate of grids for the tomography region are first compared for different combined strategies. Figs. 3 and 4 show the number of SWDs used in 10 min with a step-size of 1 h and the percentage of empty voxels based on six combined strategies for DoY 303, 2015, respectively. It can be seen from Fig. 3 that the number of SWDs used increased when the combined multi-GNSS observations are used, and the most are used with the G+C+R strategy. Fig. 4 shows that the percentage of the empty voxels is decreased as the number of satellite rays increases, but half of the voxels in the
experimental region are still not crossed by any rays. Table 2 lists the statistical results pertaining to the number of SWDs used and the percentage of empty voxels for six combined strategies. Statistical analysis reveals that the number of SWD observations is almost doubled or tripled when two or three multi-GNSS constellations are used, however, the percentage of empty voxels is not decreased as much as was expected, but only by 10–21%.

Table 2
Statistical result of the number of SWDs used and the percentage of empty voxels at DoY 303, 2015.

<table>
<thead>
<tr>
<th>System</th>
<th>Number of SWDs</th>
<th>Empty voxels (%)</th>
<th>Max.</th>
<th>Min.</th>
<th>Mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>G</td>
<td>1452</td>
<td>855</td>
<td>1075</td>
<td>73.01</td>
<td>57.95</td>
</tr>
<tr>
<td>C</td>
<td>1669</td>
<td>918</td>
<td>1233</td>
<td>83.38</td>
<td>69.74</td>
</tr>
<tr>
<td>R</td>
<td>1114</td>
<td>588</td>
<td>810</td>
<td>75.57</td>
<td>63.35</td>
</tr>
<tr>
<td>G+C</td>
<td>2716</td>
<td>2025</td>
<td>2301</td>
<td>71.02</td>
<td>55.54</td>
</tr>
<tr>
<td>G+R</td>
<td>2210</td>
<td>1493</td>
<td>1886</td>
<td>68.18</td>
<td>53.98</td>
</tr>
<tr>
<td>G+C+R</td>
<td>3638</td>
<td>2653</td>
<td>3119</td>
<td>67.33</td>
<td>52.56</td>
</tr>
</tbody>
</table>

To validate the accuracy of the established tomographic modelling using different multi-GNSS observations, the SWD residuals are compared on DoY 303, 2015. The SWD differences between different combined strategies and the PPP-estimate are first compared. Here, we compared the SWD residuals of single GPS, BDS, and GLONASS derived from different combined strategies. Figs. 5–7 show the comparison of SWD residuals distribution with elevation angle only for single GPS, BDS, and GLONASS observations, respectively. The SWD residual decreases with increasing elevation angle, which is reasonable when considering the SWD residual and elevation angle. Table 3 also presents the statistical analysis of standard errors for different systems. It is clearly that the distribution of SWD residuals with elevation angle is similar for the G, G+C, G+R, and G+C+R strategies (Fig. 5), and the standard errors are 3.98, 3.89, 3.78, and 3.78 mm, respectively. For the comparison of BDS observations (Fig. 6), the distribution of SWD residuals is resembled for the C, G+C, and G+C+R strategies with standard errors of 2.85, 2.78, and 2.75 mm, respectively. The same result can be concluded for the comparison of GLONASS data in Fig. 7 with standard errors of 3.92, 3.93, and 3.92 mm, respectively for the R, G+R, and G+C+R strategies.

In general, the accuracy of tomographic modelling is slightly improved when multi-GNSS observations are used for building the observation equation.

4.2. Comparison with radiosonde data

In this comparison, a 7-day data window is selected for the period DoY 303 to 309, 2015 to analyse the accuracy and reliability of tomographic results based on different combined multi-GNSS strategies. In this experiment, we calculated the number of SWDs and empty voxels using the observations of different combined strategies in 10 min for each step of 1 h, and then obtained the average statistical result for each day. The number of SWDs used and the percentage of empty voxels are first compared for seven days (Figs. 8 and 9, respectively). The number of satellite rays used is almost doubled or tripled for the G+C/G+R and G+C+R strategies, respectively, when compared with the single G/C/R strategies; however, the percentage of empty voxels is not decreased by much, and the largest

Fig. 5. The comparison of SWD residuals distribution with elevation angle for GPS observations derived from the PPP-estimated and tomographic results using different multi-GNSS combined strategies.
number of empty voxels was found for the C strategy while was comparable for the G+R and G+C+R strategies. This is related to the geometric structure of the BeiDou satellite constellation which consists of three kinds of satellites, including BeiDou-G satellites in the geostationary orbit (GEO), BeiDou-M satellites in medium Earth orbit (MEO), and BeiDou-I satellites in inclined geosynchronous orbits (IGSO). For the GEO satellites, they have a higher altitude (about 35,786 km), therefore, the elevation angles of BeiDou observations is higher than that of observations from other systems. For the selected region in China, the percentage of voxels crossed by rays is not only determined by the number of signals but also the geometric distribution of the signals. Statistical data pertaining to the SWDs
used and the percentage of empty voxels for the experimental period are shown in Table 4, which reveals that the SWDs used of combined strategies (G+C/G+R/G+C +R) are increased by 60–400% while the number of voxels crossed by rays is improved by 1.2–13.9%, respectively, when compared to the single G/C/R strategies.

To make a direct comparison of ZWD values derived from the radiosonde data and the different tomographic results, the ZWD values for the location of the radiosonde station are constructed using the wet refractivity profiles in the voxels obtained from the different tomographic results at UTC 00:00 and 12:00, respectively. In our study, a statistical index is introduced between the radiosonde-estimated and reconstructed ZWD values (Yao and Zhao, 2016a):

$$RE_{ZWD} = \frac{|ZWD_{RS} - ZWD_{tomo}|}{ZWD_{RS}}$$

(7)

where $RE_{ZWD}$ is the relative error of ZWD derived from radiosonde data and tomographic results based on different

<table>
<thead>
<tr>
<th>System</th>
<th>Strategy</th>
<th>G</th>
<th>C</th>
<th>R</th>
<th>G+C</th>
<th>G+R</th>
<th>G+C+R</th>
</tr>
</thead>
<tbody>
<tr>
<td>GPS</td>
<td></td>
<td>3.98</td>
<td></td>
<td></td>
<td>3.89</td>
<td>3.78</td>
<td>3.78</td>
</tr>
<tr>
<td>BDS</td>
<td></td>
<td></td>
<td>2.85</td>
<td></td>
<td>2.78</td>
<td></td>
<td>2.75</td>
</tr>
<tr>
<td>GLONASS</td>
<td></td>
<td></td>
<td></td>
<td>3.92</td>
<td></td>
<td>3.93</td>
<td>3.92</td>
</tr>
</tbody>
</table>

Table 3
Statistical result of the standard errors for different systems at Doy 303, 2015 (unit: mm).

<table>
<thead>
<tr>
<th>System</th>
<th>Strategy</th>
<th>G</th>
<th>C</th>
<th>R</th>
<th>G+C</th>
<th>G+R</th>
<th>G+C+R</th>
</tr>
</thead>
<tbody>
<tr>
<td>GPS</td>
<td></td>
<td>1252</td>
<td>1134</td>
<td>1193</td>
<td>69.03</td>
<td>66.76</td>
<td>67.78</td>
</tr>
<tr>
<td>BDS</td>
<td></td>
<td>1303</td>
<td>1172</td>
<td>1245</td>
<td>77.84</td>
<td>74.64</td>
<td>76.56</td>
</tr>
<tr>
<td>GLONASS</td>
<td></td>
<td>880</td>
<td>686</td>
<td>809</td>
<td>73.08</td>
<td>66.40</td>
<td>69.46</td>
</tr>
<tr>
<td>G+C</td>
<td></td>
<td>2555</td>
<td>2344</td>
<td>2438</td>
<td>67.61</td>
<td>65.63</td>
<td>66.60</td>
</tr>
<tr>
<td>G+R</td>
<td></td>
<td>2131</td>
<td>1881</td>
<td>2002</td>
<td>65.98</td>
<td>60.30</td>
<td>63.06</td>
</tr>
<tr>
<td>G+C+R</td>
<td></td>
<td>3434</td>
<td>3087</td>
<td>3247</td>
<td>65.41</td>
<td>60.16</td>
<td>62.66</td>
</tr>
</tbody>
</table>

Table 4
Statistical result of the number of SWDs and the empty voxels.

<table>
<thead>
<tr>
<th>Number of SWDs</th>
<th>Empty voxels (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Max.</td>
<td>Min.</td>
</tr>
<tr>
<td>G</td>
<td>1134</td>
</tr>
<tr>
<td>C</td>
<td>1172</td>
</tr>
<tr>
<td>R</td>
<td>686</td>
</tr>
<tr>
<td>G+C</td>
<td>2344</td>
</tr>
<tr>
<td>G+R</td>
<td>1881</td>
</tr>
<tr>
<td>G+C+R</td>
<td>3087</td>
</tr>
</tbody>
</table>

Fig. 8. Observed GNSS observations derived from GPS, BDS and GLONASS.

Fig. 9. Voxels without any satellite rays crossed SWD observations.
combined strategies, $ZWD_{RS}$ are the ZWD values calculated based on radiosonde values while $ZWD_{Tomo}$ is reconstructed from the tomographic results. Fig. 10 shows the comparison of relative error of ZWD derived from radiosonde-estimated and tomographic results based on the various combined strategies for the period DoY 303 to 309, 2015. It can be seen that the relative errors in ZWD for different tomographic results at the same epochs vary over time, and the statistical result revels that the average relative errors for six combined strategies during the test period are 9.36%, 9.58%, 10.26%, 9.60%, 9.83%, and 9.87%, respectively.

One point should be noted that the ZWD is the integrated value of wet refractivity along the zenith directions from the surface to the tomographic top boundary, which cannot reflect the wet refractivity profile information: if two vertical layers are arbitrarily exchanged, the ZWD value remains the same but the vertical structure has changed significantly. Therefore, the wet refractivity profile information is further compared with radiosonde data for the seven days at UTC 00:00 and 12:00, respectively. Two groups of wet refractivity profiles at specific epochs UTC 00:00, DoY 304 and UTC 00:00, DoY 307, are shown respectively (Fig. 11). Those two epochs are selected as they correspond to the minimum and maximum relative ZWD values. It can be seen from Fig. 11 that the wet refractivity profiles derived from radiosonde data and various tomographic result agree with each other, and the wet refractivity, over the two epochs, shows a negative exponential trend with altitude. For Fig. 11(a), the reconstructed wet refractivity changed significantly, especially in the bottom layers while the tomographic results for different strategies match the radiosonde profiles in Fig. 11(b). The numerical result shows that the RMS error of wet refractivity profiles...
from the six combined strategies are 7.33, 6.12, 7.37, 6.96, 7.35, and 7.02 ppm and 11.51, 11.20, 12.35, 11.61, 12.01, and 11.87 ppm, respectively. Fig. 12 shows the comparison of RMS differences between the radiosonde and tomographic wet refractivity using different combined multi-GNSS observations for the selected period at UTC 00:00 and 12:00. From Fig. 12 it was concluded that the RMS errors of wet refractivity derived from the combined strategies (G+C, G+R, and G+C+R) were not always superior to that of the single strategies (G, C, and R). Although multi-GNSS can provide more observations for tomographic modelling, it cannot improve the quality of the tomography as was expected. The average RMS errors arising during the test period (Table 5) reveal that the accuracy of RMS is improved by between 0 and 5.5% when using multi-GNSS observations.

To investigate the relationship of wet refractivity with respect to altitude, the RMS and relative error of wet refractivity derived from radiosonde data and tomographic results based on different combined strategies are compared. Fig. 13 shows the changes of RMS and relative error

<table>
<thead>
<tr>
<th>RMS (ppm)</th>
<th>Max.</th>
<th>Min.</th>
<th>Mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>G</td>
<td>10.38</td>
<td>6.90</td>
<td>9.15</td>
</tr>
<tr>
<td>C</td>
<td>12.59</td>
<td>7.09</td>
<td>9.35</td>
</tr>
<tr>
<td>R</td>
<td>11.82</td>
<td>7.31</td>
<td>9.31</td>
</tr>
<tr>
<td>G+C</td>
<td>11.51</td>
<td>7.13</td>
<td>9.22</td>
</tr>
<tr>
<td>G+R</td>
<td>9.93</td>
<td>7.20</td>
<td>8.84</td>
</tr>
<tr>
<td>G+C+R</td>
<td>10.46</td>
<td>7.48</td>
<td>8.93</td>
</tr>
</tbody>
</table>

Fig. 12. Comparison of RMS difference between the radiosonde and tomographic wet refractivity using different combined GNSS observation.

Fig. 13. RMS and relative error change with altitude for different tomographic results based on different combined strategies for the experimental period.
of wet refractivity with altitude for different tomographic results for the selected seven-day period. It can be seen that the RMS error is slightly lower for the G+R/G+C+R strategies while values for the C/R strategies are poor at lower layers. Generally, the RMS error is decreased with height while the relative error shows the opposite trend. The main reason for this is that the wet refractivity is very large in the lower layers which leads to a large RMS error and a small relative error: the values in upper layers are relatively small, and even a small discrepancy can cause a large relative error.

5. Conclusion and discussion

A GNSS tropospheric tomography comparison has been carried out to investigate the reconstruction of water vapour fields when introducing multi-GNSS observations. Six kinds of combined strategies have been selected: GPS, BDS, GLONASS, GPS+BDS, GPS+GLONASS, and GPS+BDS+GLONASS. The tomographic experiments were performed using the 7-day data from a CORS Network at Guiyang, China, based on the different combined strategies. Horizontal and vertical constraints are also imposed on the tomographic modelling, and the tomographic top boundary is determined based on the distribution of water vapour with altitude from the COSMIC RO “wetPrf” profiles.

A comparison of tomographic results for the experimental period shows that the number of satellite rays used has been almost doubled or tripled when the two or three kinds of multi-GNSS observations were used in the tomographic modelling, but the percentage of empty voxels did not decreased as much as was expected (only by about 1.2–13.9%). The comparison of wet refractivity profile derived from raiddosonde data and tomographic results based on various combined strategies revealed that the accuracy of RMS for the reconstructed wet refractivity field is only improved by 0–5.5%, therefore, the reconstructed quality of three-dimensional water vapour fields was improved, but not by much, when the multi-GNSS observations are used for establishing the observation equation using the current tomographic technique.

Some possible reasons may explain these results: the spatial resolution of the tomographic modelling was not improved as was expected. As presented in Section 4, there remained some 50% of the voxels uncrossed by any satellite rays even when data from the GPS+BDS+GLONASS strategy were used. Another possible reason was that the increased number of observations may have no positive effect on the tomographic resolution, especially for those additional multi-GNSS rays crossing voxels which have been penetrated by the SWDs from the single GPS/BDS/GLONASS observations. In addition, the satellite rays crossing outwards from the side face of the research area are not treated as conveying effective information: such rays are also important when improving coverage rates in the tomography area, especially among those voxels in the lower layers. Therefore, improving the spatial resolution of the tomographic modelling is a key factor, which can be addressed by increasing the spatial density of ground-based GNSS stations (Adavi and Mashhadi-Hossaini, 2014; Benevides et al., 2016) and using the rays crossing outwards from the side face of the research area (Yao and Zhao, 2016a,b). In addition, station distribution, the horizontal resolution defined for the study area and the proximity of the stations to the voxels border also very important for the tomography result (Bender and Raabe, 2007; Benevides et al., 2016). After those problems are overcome, the quality of reconstruction of multi-GNSS tropospheric tomography is expected to be improved significantly.

Acknowledgement

The authors would like to thank the Guiyang Institute of Surveying and Mapping and International GNSS Service (IGS) for providing the multi-GNSS observations and the post-mission satellite precise products. The IGRA and ECMWF are also thanked for providing the corresponding meteorological parameters. The University Corporation for Atmospheric Research (UCAR) is thanked for providing the COSMIC RO products. This research was supported by the Excellent Youth Science and Technology Fund Project of Xi’an University of Science and Technology (2018YQ3-12) and the Startup Foundation for Doctor of Xi’an University of Science and Technology “Research on the troposphere tomography and its application on severe weather warnings (2017QDJ041)”).

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